

Minnesota State High School Mathematics League

Individual Event

2004-05 Event 5A

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 20 minutes for this event.

- _____ 1. If the integers 1, 2, ... 8 are entered into the eight squares in Figure 1 so that successive integers are not in squares that are adjacent or have a common corner, then what is the sum of the four integers in the middle column?

- _____ 2. 99% of the weight of 100 pounds of potatoes is water. An overnight drying process reduces the amount of water so it constitutes only 98% of the weight of the potatoes. What do the potatoes then weigh?

- _____ 3. New year's Day in 2005 was on a Saturday. The grid in Figure 3 is torn from a January, 2005 calendar. The dates in the marked squares sum to 50. What three days of the week are represented in the part of the calendar shown?
 A = _____
 B = _____
 C = _____

- _____ 4. Racing to see who can be the first to swim 10 laps, Allison and Beth start at opposite ends of a pool. Each girl swims steadily at her own rate. They first meet at a point a yards from the side where Allison began, and they next meet b yards from the side where Beth began. What, in terms of a and b , is the length of the pool?

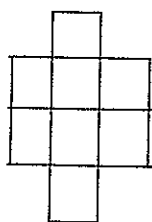


Figure 1

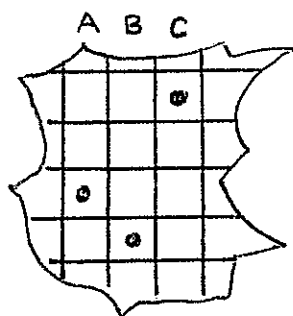
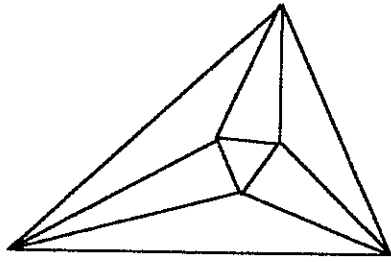


Figure 3

Name _____ Team _____



Minnesota State High School Mathematics League Individual Event

2004-05 Event 5B

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

- _____ 1. Find the area of the trapezoid (Figure 1) having bases of 5 and 13, and a height of 6.
- _____ 2. Using the tape used to cordon off an area, police need 3 rolls to enclose a square containing 900 square yards. What (in square yards) will be the area of a square that can be enclosed using 4 rolls?
- _____ 3. A regular hexagon with area I is inscribed in a circle. At each vertex, a line segment tangent to the circle is drawn. These line segments form a second regular hexagon with area C circumscribing the circle. Find the ratio $\frac{C}{I}$.
- _____ 4. The base of a right circular cone has radius R . The top of the cone is cut off by a plane parallel to and h units above the plane of the base. The resulting solid, called the frustum of a cone, has a circular top of radius $r < R$. Express the volume of a frustum of a cone in terms of r , R and h .

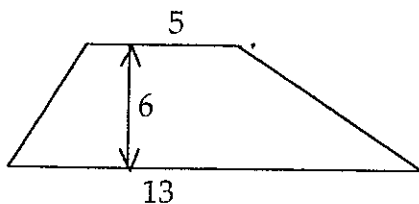
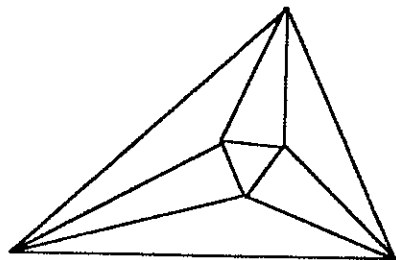


Figure 1

Name _____ Team _____



Minnesota State High School Mathematics League

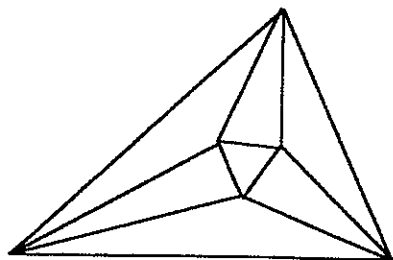
Individual Event

2004-05 Event 5C

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

- _____ 1. Two ordinary dice are rolled. What is the probability that the total showing on the dice will be 10 or greater?
- _____ 2. From 7 applicants for a job, the personnel department of a company is to present to management in alphabetical order a list of 3 suitable candidates. How many different lists are possible?
- _____ 3. How many even three digit numbers can be formed using digits from the set $\{0, 1, 2, 3, 5, 6, 7\}$ if all digits of the integer must be distinct?
- _____ 4. In how many ways can 20 identical balls be distributed into six cans labeled A, B, C, D, E, F? Three ways, for example, would be $\{5, 5, 4, 3, 2, 1\}$, $\{4, 5, 5, 3, 2, 1\}$ and $\{9, 5, 0, 1, 2, 3\}$.

Name _____ Team _____



Minnesota State High School Mathematics League

Individual Event

2004-05 Event 5D

Questions in this event are written, with permission, as variations of problems from the 2004 AMC-12 Exam. Review of this exam is excellent preparation, not only for this event, but for the 2005 AMC-12, which we strongly encourage you to take.

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

- _____ 1. What will be the result if the sum of the first 2004 even counting numbers (that is, the first 2004 even positive integers) is subtracted from the sum of the first 2005 odd counting numbers?

- _____ 2. What is the median of 125 consecutive integers if their sum is 10,000?

- _____ 3. Rectangle ABCD (Figure 3) has sides of length $AB = 4$, $BC = 7$. A semi-circle is constructed inside the rectangle using AB as a diameter, and a line drawn from C tangent to the semi-circle intersects AD at E . What is the length of CE ?

- _____ 4. Figure 4 shows a view from the top of four spheres, each with radius 1, resting on a horizontal plane P so that their centers form a square of side length 2. Suppose a sphere of radius 2 is placed on top of them so it is tangent to all four of the smaller spheres. What is the distance from the plane P to the top of the larger sphere?

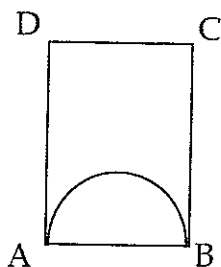


Figure 3

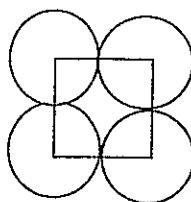
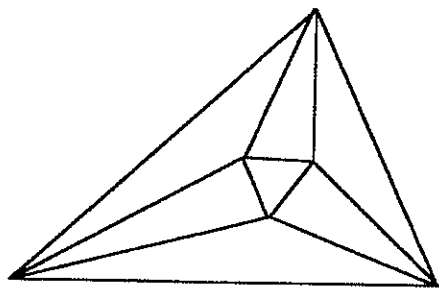


Figure 4

Name _____ Team _____



Minnesota State High School Mathematics League

Team Event

2004-05 Meet 5

Each question is worth 4 points. Team members may cooperate in any way, but at the end of twenty minutes, one set of answers is to be submitted. Put answers on the lines provided.

- Let A, B, C , and D be four points on a line with $AB = CD$ (Figure 1). Semi-circles are drawn above the line with diameters $AD = 2R$ and $BC = 2r$, and semi-circles with diameters of AB and CD are drawn below the line. If $R = 2m + 1$ and $r = 2n + 1$ are both odd integers, then the area of the region enclosed by the four semi-circles is an integral multiple of π . Express this integer in terms of m and n .

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- In Figure 2, $AE \perp AB$, $BC \perp AB$, $AB = 6$, $BC = 7$, and $AE = 11$. Find $\text{Area}(\triangle ADE) - \text{Area}(\triangle BCD)$.
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- Let the arbitrary $\triangle ABC$ with sides a, b , and c (Figure 3) have area T . On each side of the triangle, construct an equilateral triangle having, respectively, areas T_a, T_b , and T_c . If $\angle A = \frac{\pi}{3}$, a simple equation relates T, T_a, T_b , and T_c . Find it.

$T =$

- A sudden rainstorm forced us to abandon our backyard picnic. When we came out the next morning, we found that an empty salad bowl, left behind in our dash to the house, had 4.5 inches of rainwater in it. Wondering how many inches of rain had fallen, we noted that the bowl was shaped like the frustum of a right circular cone, having a base with a diameter of 6 inches, a top diameter of 14 inches, and a depth of 6 inches. How many inches of rain fell; i.e. how many inches would have stood in a right circular cylinder if there had been one on the table?
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- Starting at diametrically opposite points, Allison and Beth run in opposite directions on a circular track, each girl running steadily at her own rate. They first meet after Allison has run 150 yards. They next meet after Beth has run 210 yards past their first meeting point. What is the length of the track in yards?

Team _____

6. Right $\triangle ABC$ has legs of lengths 3 and 8. The long leg is used as the diameter of a circle that cuts the hypotenuse at D (Figure 6). A line tangent to the circle at D cuts AC at E . What is the length of CE ?

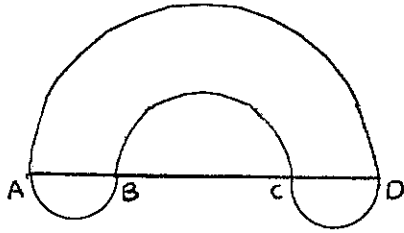


Figure 1

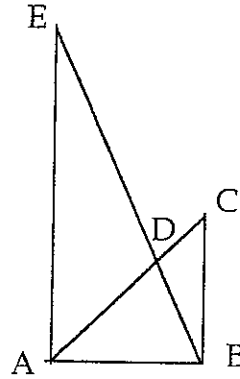


Figure 2

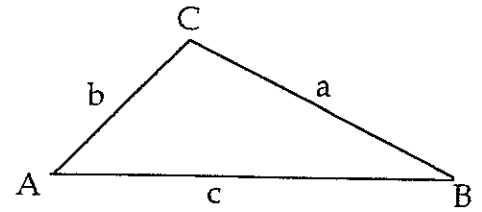


Figure 3

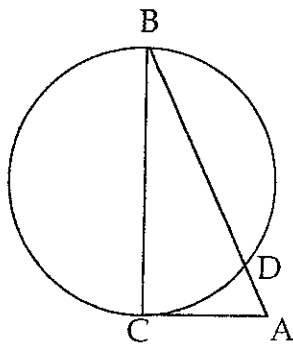


Figure 6

Team _____